Mathematics Interventions & Algebra Readiness: Best Evidence from Scientific Research and Research Mathematicians

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Mathematics is amazingly compressible: you may struggle a long time, step by step, to work through the same process or idea from several approaches. But once you really understand it and have the mental perspective to see it as a whole, there is often a tremendous mental compression. You can file it away, recall it quickly and completely when you need it, and use it as just one step in some other mental process. The insight that goes with this compression is one of the real joys of mathematics.

** From David Foster
Silicon Valley Math Initiative

W. T. Thurston

Overview

- What to teach
  (highlights from publicly released material from Presidentially appointed National Math Advisory Panel)
- How to teach (findings from meta analysis work of Center On Instruction/ RG Research Group and parallel work from National Math Advisory Panel
- A few details on interventions (Brad Witzel) – teaching fractions through explicit language and CRA procedures

What to Teach #1: Fractions/ Rational Number

For students to be algebra ready, they must really learn and master concepts and procedures related to the rational number/fractions

Evidence Base: Moderate

Sources: Descriptive research and expert opinion from research mathematicians.

The Research Base

- Fourth graders who are weak in math (based on achievement test) are weakest in:
  - Knowing where to put fractions on a number line/ Magnitude comparison (e.g Is 8/9 bigger than 9/11?)
  - Matching visual representations to actual fractions

(S. Hecht, 2007)

The Focus on Fractions

- The National Mathematics Panel suggests an increased emphasis on fractions computation in grades 4-8
- Teaching the foundations for success in algebra, “middle school students should have a thorough understanding of positive and negative fractions on a number line; represent and compare fractions, decimals, and related percents; and estimate their size” (NMP, 2008, 18).

What to Teach (1): Fractions

1. Many meanings of fraction: though all fit one definition
   - Part of whole (unit) or several units
   - Part of a set
   - A precise point on number line
   - Mathematically equivalent to division

Fractions:

2. Corollary: don’t say a fraction is a part of a whole since that is not correct.
3. Fractions need to taught as a solid unit.
4. Use of Concrete objects: Expeditious, quickly link to visual representations. Major emphasis on visual representations to abstract.

Fractions:

5. Computations involving fractions require a good deal of instructional time and practice.
6. Use of number line essential.

Problem: Number line**

Where are a+b, b-a and a-b?

What can you say about where a/b is?

What to teach (2): Additional Work on Proportion, Ratio, Decimals, & Decimal Fractions.

1. Consistent focus on number line
2. Mix of word problems and calculations

Always, Sometimes, or Never True

A.

If you double the numerator of a fraction, you double the size of the fraction.

$$\frac{a}{b} \longrightarrow \frac{2 \times a}{b}$$

What Else to Stress so Students are Algebra Ready

What to teach (3):

1. Number bonds/families (especially those based on 10 (8+2 or 3+7)
2. Fluency with commutatively, associative, and distribute law (composition & decomposition, e.g. 7+5: find number familiarity for 10 is 7+3 so we decompose the 5)
3. Fluency with basic facts using both rote and number families
4. Reciprocal relationship between algorithmic and quick fact retrieval

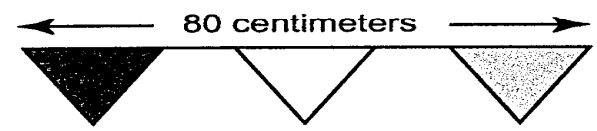
What to teach (4): Basic Concepts of Measurement & Geometry

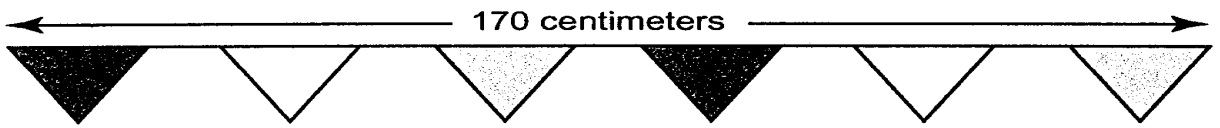
1. Good benchmark for algebra readiness is understanding of similar triangles.

Party Flags

This problem gives you the chance to:

- find sizes by interpreting a diagram
- express a function by a formula





Erica is putting up lines of colored flags for a party.

The flags are all the same size and are spaced equally along the line.

1. Calculate the length of the sides of each flag, and the space between flags.

Show all your work clearly.

2. How long will a line of *n* flags be?

Write down a formula to show how long a line of *n* flags would be.

** From David Foster
Silicon Valley Math Initiative
Algebra Forum Online Event 10/23

Poll: How did you solve question #1 of Party Flags?

A. I looked at one set of flags used guess and check and figured the flag length was 20 and the string 10.

B. I looked and both sets of flags and used a process where I double the first set of flags, compared it to the second set of flags and found that the string must be 10. Then I was able to go back and find the flag length was 20.

C. I set up two equations with two unknowns and solve it using simultaneous equations.

The Findings from Party Flags

- The task may be approached as a system of simultaneous equations, almost no algebra students used such an approach.
- 49% of algebra students had no success.
- 44% accurately found the two lengths (most commonly by an estimation strategy only using one constraint).
- 21% correctly used both constraints (the length of three flags is 80 cm. and the length of 6 flags is 170 cm.).
- 7% of the students were able to develop a valid generalization for n flags.

**MAC Final Report 2001

Any questions?